UNIT – V

9. (a) State Rank-Nullity Theorem.

   (b) Define the basis of a vector space.

   (c) Define similar matrices.

   (d) Define annihilator of subspace w of a vector space $V(F)$.

   (e) Define critical points.

   (f) Determine the stereographic projection of $z = 1 - i$ on the sphere of radius $\frac{1}{2}$ and centre $\left(0, 0, \frac{1}{2}\right)$.

Roll No. ....

60578

B. Sc. 6th Semester Maths (Hons.)

Examination – April, 2017

REAL AND COMPLEX ANALYSIS

Paper : BHM-361

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *one* question from each Unit. Question No. 9 is *compulsory*.

UNIT – I

1. (a) If $u = \frac{x}{y-z}, v = \frac{y}{z-x}, w = \frac{z}{x-y}$, verify that :

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$$

   (b) To show that :

$$B(m,n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}, \text{ if } m, n \in z^+$$

2. (a) Prove that :

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

(b) $\iiint(ax+by+cz)\,dx\,dy\,dz$ over ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$.

UNIT – II

3. (a) Find the Fourier series expansion of :

$$f(x)=\frac{1}{4}(\pi-x)^2,\ 0<x<2\pi$$

(b) Find the Fourier series for $f(x)=e^{-x}$ in $(0, 2\pi)$.

4. (a) Find the Fourier expansion for the function $f(x)=x-x^2, -1<x<1$.

(b) Obtain $f(x) = x$ as half range cosine series as $0<x<2$.

UNIT – III

5. (a) Show that the function :

$$f(z)=\begin{cases}\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & \text{when } z\neq 0\\ 0 & z=0\end{cases}$$

is continuous and the C – R equations are satisfied at the origin, yet $f'(0)$ does not exist.

(b) If $w = f(z)$ is a regular function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f'(z)|=0$$

6. (a) Show that $u(x,y)=e^{-x}(x\sin y-y\cos y)$ is harmonic and find $v(x, y)$ such that $f(z) = u + iv$ is analytic.

(b) Find analytic function $f(z) = u(r, \theta) + i\, v(r, \theta)$ such that :

$$v(r,\theta)=r^2\cos 2\theta-r\cos\theta+2$$

UNIT – IV

7. (a) Find the image of $|z-3i| = 3$ under the mapping $w = 1/z$.

(b) Find the fixed points, normal form and nature of bilinear transformation $w=\dfrac{z}{z-2}$.

8. (a) Show that cross ratio remains invariant under Mobius Transformation.

(b) Show that the transformation $w=\tan^2\left(\frac{\pi\sqrt{2}}{4}\right)$ map the interior of the unit circle $|w|=1$ into the interior of the parabola.

8. (a) Every open continuous image of a second countable space is second countable.

(b) Discuss whether the following topological spaces defined on $X = \{a, b, c\}$ are τ_2-spaces

(i) $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$

(ii) $\tau_2 = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$

UNIT – V

9. (i) List all the possible topologies of the set $X = \{a, b\}$.

(ii) Define Lower limit topology.

(iii) Define To Space.

(iv) Define interior of a set.

(v) Define Connected & Disconnected set.

(vi) State Lindelof Theorem.

Roll No. ....

60581

B. Sc. 6th Semester (Math) (Hons.) Examination – April, 2017

CHEMISTRY TOPOLOGY

Paper : BHM-364

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***one*** questions from each Unit. Question No. 9 is ***compulsory***.

UNIT – I

1. (a) Find all possible topologies for the set $x = \{a, b, c\}$.

(b) Prove that every family of topologies on a set has greater lower bound.

2. (a) Prove that intersection of two topologies is also topology, but Union of two topologies is not necessary a topology.

(b) Let (X, τ) be a topological space and let $A, B, \subseteq X$. Then

(i) $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$

(ii) $(A \cap B)^\circ = A^\circ B^\circ$

UNIT – II

3. (a) Give an example of two topological space X and Y and a mapping $f : x \to y$ which is

(i) Open mapping but not closed mapping

(ii) Closed mapping but not open mapping

(iii) Both open as well as closed mapping

(iv) Neither open nor closed mapping

(b) Let (X, τ) and (Y, β) be topological spaces and let the mapping $f : X \to Y$ be one-one, onto. Then f is homeomorphism iff.

$f(A^\circ) = (f(A))^\circ$ for every subset A of X.

4. (a) A space (X, τ) is disconnected iff $\exists A \neq \phi, X$ and is open as well as closed set.

(b) The closure of a connected set is connected set in a topological space.

UNIT – III

5. (a) Prove that usual topological space is not compact.

(b) Every compact subset of a Hausdorff space is close.

6. (a) Every compact topological space X is locally compact.

(b) Contineuous Images of a compact set is a compact set.

UNIT – IV

7. (a) Discrete topological space on uncountable set is not second countable space.

(b) Prove that every metric space is first countable space.

8. (a) State and prove Blasius theorem.

(b) Discuss about Image of a source with regard to a circle.

SECTION – V

9. (a) What is significance of the equation of continuity.

(b) Define Irrotational motion in two-dimensions.

(c) Define Lagrange's stream function.

(d) Define sources and sinks in two-dimensional.

(e) State Bernoulli's equation.

(f) Find the equation of the stream lines for the flow $q = -3y^2 i - 6x\bar{j}$ at the point (1, 1).

Roll No. ....

60582

B. Sc. 6th Semester (Math) (Hons.) Examination – April, 2017

FLUID MECHANICS

Paper : BHM-365

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***one*** questions from each Section. Question. No. 9 is ***compulsory***.

SECTION – I

1. (a) The velocities at a point in a fluid in the Eulerian system are given by $u = x + y + z + t$, $v = 2(x + y + z) + t$ $w = 3(x + y + z) + t$. Obtain the displacements of a fluid particle in the Lagrangian system.

(b) Determine the stream line and path of the particle when $u = \frac{2xt}{1+t^2}, v = \frac{2yt}{1+t^2}, w = \frac{2zt}{1+t^2}$.

2. (a) The equation of continuity in cylindrical co-ordinates.

(b) The velocity in the flow field is given by

$q = i\,(Az - By) + j\,(Bx - Cz) + k\,(Cy - Ax)$

where A, B, C are non-zero constants. Determine the equations of the vortex lines.

SECTION – II

3. (a) Components of acceleration in spherical polar co-ordinates (r, θ, ϕ) with velocity components (v_r, v_θ, v_ϕ).

(b) Derive Euler's equation of motion in cylindrical coordinates.

4. (a) Obtain general equation of motion for Impulsive motion.

(b) State and prove Bernoulli's Theorem (steady motion with no velocity potential and conservative field of force)

SECTION – III

5. (a) State and prove Kelvin's minimum energy theorem.

(b) Acyclic irrotational motion is impossible in a liquid bounded entirely by fixed rigid walls.

6. (a) Discuss motion of a sphere through an infinite mass of a liquid at rest at infinity.

(b) Discuss about image of a three-dimensional source with regard to a sphere.

SECTION – IV

7. (a) The velocity potential function for a two-dimensional flow is $\phi = x\,(2y - 1)$. At a point $P(4, 5)$ determine.

(i) the velocity stream

(ii) the value of function

(b) Find the stream function of two-dimensional motion due to two equal sources and an equal sink situated midway between them.

(b) A particle describes the equiangular spiral $r = ae^{\theta \cot \alpha}$ under a force to the pole. Find the law of force.

8. (a) To find the acceleration of a particle in terms of cylindrical polar coordinates.

(b) Find the components of acceleration in terms of spherical polar coordinates.

SECTION – V

9. (a) Define Apse and Apsidal distances.

(b) What is horizontal range of a projectile.

(c) Define angular acceleration along a plane curve.

(d) A body of mass 25 gms is acted upon by a constant force. It acquires a velocity of 2 cm/sec in 5 seconds from rest. Find how large is the force acting ?

(e) Write down Kepler's law of planetary motion.

(f) Define S. H. M. and its amplitude.

Roll No. ....

60580

B. Sc. 6th Semester (Mathematics) (Hons.) Examination – April, 2017

DYNAMICS

Paper : BHM-363

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 9 of Section-**V** is *compulsory*.

SECTION – I

1. (a) A particle describes a curve for which S and ψ vanish simultaneously with uniform speed V. If acceleration at any point be $\dfrac{V^2 C}{S^2 + C^2}$, prove that the curve is a catenary.

(b) A particle moves along the curve $x = 4t$, $y = 6t - t^2$, find the tangential and normal acceleration at $t = 3$.

2. (a) To obtain the expressions for velocity and position of a particle executing simple harmonic motion.

(b) At the end of three successive seconds, the distances of moving point with SHM from the mean position measured in the same direction are 1, 5 and 5. Show that the time of one oscillation is $\frac{2\pi}{\theta}$ where $\cos\theta = \frac{3}{5}$.

SECTION – II

3. (a) Two scale pans each of mass 4 kg are connected by a light spring passing over a pulley. Show how to divided a mass of 10 kg in two scale pans, so as to produce an acceleration of $g/9$.

(b) A mass of 3 kg descending vertically draws up a mass of 2 kg means of light string passing over a smooth pulley. At the end of 5 second string breaks. Find how much higher the 2 kg mass will go.

4. (a) State and prove the principle of work and energy.

(b) A mass of 10 kg falls 10 m and is brought to rest be penetrating 1 m into the sand. Find the resistance of the sand.

SECTION – III

5. (a) A particle slides down the outside of a smooth vertical circle starting from rest at the highest point. Discuss the motion.

(b) Two particles are let drop from the cusp of a cycloid down the curve at an interval of time t. Prove that They will meet in time $2\pi\sqrt{\frac{a}{g}} + \frac{t}{2}$.

6. (a) A body is project at an angle α to the horizon so as to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to :

$$2a\cot\frac{\alpha}{2}$$

(b) A body thrown in a direction making an angle of 30° with the horizon passes through a point 400 $\sqrt{3}$ ft horizontally from the point of projection and 76 ft above it. Find the velocity of projection.

SECTION – IV

7. (a) Derive differential equation of central orbit in polar form.

(b) Let V be an inner product space over $F(=\mathbb{C})$ and $T \in L(V)$ be such that $\|Tv\| = \|v\|, \forall v \in V$. Then show that T is unitary. 6

SECTION – V

9. (a) Let F be a subfield of the field K. Then show that K can be made into a vector space over F. 2

(b) Let W be subspace of a vector space $V(F)$. Prove that : 2

$$\dim(V/W) = \dim V - \dim W$$

(c) Find the range space and null space of the linear transformation $T = \mathbb{R}^2 + \mathbb{R}^2$ defined by $T(x, y) = (x + y, x)$. 2

(d) Find matrix of the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2, 7x_2)$ related to the standard basis of $\mathbb{R}^2$ and $\mathbb{R}^3$. 2

(e) Normalise the vectors $u = (1, 2, -1)$, $v = (2, 1, -4)$, $w = (2, 1, -4)$. 2

(f) Let $u, v \in V$ and V is an inner product space over (F). Prove that : 2

$$(\alpha u + \beta v, \alpha u + \beta v) = |\alpha|^2 \|u\|^2 |\beta|^2 \|v\|^2 + 2\mathrm{Re}\left(\alpha\overline{\beta}(u, v)\right)$$

where $\alpha, \beta \in F$.

Roll No. ....

60579

B. Sc. 6th Semester (Mathematics) (Hons.) Examination – April, 2017

LINEAR ALGEBRA

Paper : BHM-362

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section-V) is ***compulsory***.

SECTION – I

1. (a) Let V be a vector space over $\mathbb{R}^3$. Whether $W = \{(x, y, z) : x = y - z \text{ and } 2x + 3y - z = 0\}$ is a subspace of V or not. 6

(b) Let $v_1 = (2, -1, 0)$, $v_2 = (1, 2, 1)$, $v_3 = (0, 2, -1)$, show that v_1, v_2, v_3 are linearly independent. Express the vectors $(3, 2, 1)$, $(1, 1, 1)$ as a linear combination of v_1, v_2, v_3. 6

2. (a) If $S = \{v_1, v_2, ...v_3\}$ spans a vector space $V(F)$ then show that there exist on subset of S which is a basis of V. 6

(b) Determine a basis and dimensions of the subspace W of $\mathbb{R}^3(R)$ where : 6

$$W = \{(u_1, u_2, u_3) : u_i \in R,\ 2u_1 + u_2 - 3u_3 = 0\}$$

SECTION – II

3. (a) Two finite dimensional vector space over the same field are isomorphic if and only if they have same dimension. Prove it. 6

(b) Let the transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(X) = AX$ where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$. Find the linear transformation and hence find image of $X = (2, 0, 5)$.

4. (a) If $T : U(F) \to V(F)$ is a linear transformation then show that : 6

$$\text{Rank}(T) + \text{Nullity}(T) = \dim U$$

(b) Find the linear transformation $T = \mathbb{R}^3 \to \mathbb{R}^4$ whose null space is generated by (0, 1, –3) and (0, –3, 4). 6

SECTION – III

5. (a) Show that a linear transformation $T : U \to V$ is non-singular iff T is one to one. 6

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}. 6

6. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation define by $T(u, v, w) = (3u, u - v, 2u + v + w)$. Prove that $(T^2 - I)(T - 3I) = 0$. Find the minimal polynomial of T. 6

(b) Find the matrix of transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (2x_1 + x_2, 2x_2 - x_1)$ relative to the basis B {(1, 1, 0), (1, 0, 1), (0, 1, 1)} and B' = {(1, 1), (1, –1)} of $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. 6

SECTION – IV

7. (a) State and prove Schwarz's inequality for inner product space. 6

(b) Prove that every inner product space is a metric space. 6

8. (a) Let W be a subspace of finite dimensional inner product space V and $W^{\perp}$ be the orthogonal complement of W. Then show that :

$$V = W \oplus W^{\perp}$$

UNIT – V

9. (a) State Rank-Nullity Theorem.

(b) Define the basis of a vector space.

(c) Define similar matrices.

(d) Define annihilator of subspace w of a vector space $V(F)$.

(e) Define critical points.

(f) Determine the stereographic projection of $z = 1 - i$ on the sphere of radius $\frac{1}{2}$ and centre $\left(0, 0, \frac{1}{2}\right)$.

Roll No. ....

60578

B. Sc. 6th Semester Maths (Hons.)

Examination – April, 2017

REAL AND COMPLEX ANALYSIS

Paper : BHM-361

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *one* question from each Unit. Question No. 9 is *compulsory*.

UNIT – I

1. (a) If $u = \frac{x}{y-z}, v = \frac{y}{z-x}, w = \frac{z}{x-y}$, verify that :

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$$

(b) To show that :

$$B(m,n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}, \text{ if } m, n \in z^+$$

2. (a) Prove that :

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

(b) $\iiint (ax + by + cz)\, dx\, dy\, dz$ over ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$.

UNIT – II

3. (a) Find the Fourier series expansion of :

$$f(x) = \frac{1}{4}(\pi - x)^2,\ 0 < x < 2\pi$$

(b) Find the Fourier series for $f(x) = e^{-x}$ in $(0, 2\pi)$.

4. (a) Find the Fourier expansion for the function $f(x) = x - x^2, -1 < x < 1$.

(b) Obtain $f(x) = x$ as half range cosine series as $0 < x < 2$.

UNIT – III

5. (a) Show that the function :

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} & \text{when } z \ne 0 \\ 0 & z = 0 \end{cases}$$

is continuous and the C – R equations are satisfied at the origin, yet $f'(0)$ does not exist.

(b) If $w = f(z)$ is a regular function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f'(z)| = 0$$

6. (a) Show that $u(x, y) = e^{-x}(x \sin y - y \cos y)$ is harmonic and find $v(x, y)$ such that $f(z) = u + iv$ is analytic.

(b) Find analytic function $f(z) = u(r, \theta) + i\, v(r, \theta)$ such that :

$$v(r, \theta) = r^2 \cos 2\theta - r \cos\theta + 2$$

UNIT – IV

7. (a) Find the image of $|z - 3i| = 3$ under the mapping $w = 1/z$.

(b) Find the fixed points, normal form and nature of bilinear transformation $w = \frac{z}{z-2}$.

8. (a) Show that cross ratio remains invariant under Mobius Transformation.

(b) Show that the transformation $w = \tan^2\left(\frac{\pi\sqrt{2}}{4}\right)$ map the interior of the unit circle $|w| = 1$ into the interior of the parabola.